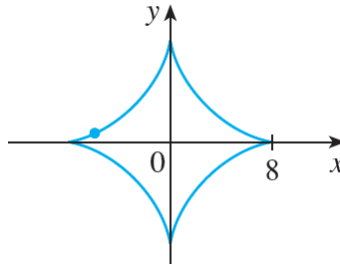


Exercise 30

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$x^{2/3} + y^{2/3} = 4, \quad (-3\sqrt{3}, 1), \quad (\text{astroid})$$



Solution

The aim is to evaluate y' at $x = -3\sqrt{3}$ and $y = 1$ in order to find the slope there. Differentiate both sides of the given equation with respect to x .

$$\frac{d}{dx}(x^{2/3} + y^{2/3}) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}(x^{2/3}) + \frac{d}{dx}(y^{2/3}) = 0$$

$$\left(\frac{2}{3}x^{-1/3}\right) + \left[\frac{2}{3}y^{-1/3} \cdot \frac{d}{dx}(y)\right] = 0$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$x^{-1/3} + y^{-1/3}y' = 0$$

Solve for y' .

$$y' = -\sqrt[3]{\frac{y}{x}}$$

Evaluate y' at $x = -3\sqrt{3}$ and $y = 1$.

$$y'(-3\sqrt{3}, 1) = -\sqrt[3]{\frac{1}{-3\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

Therefore, the equation of the tangent line to the curve represented by $x^{2/3} + y^{2/3} = 4$ at $(-3\sqrt{3}, 1)$ is

$$y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3}).$$

Below is a graph of the curve and the tangent line at $(-3\sqrt{3}, 1)$.

